Toward a decompositional approach to diagnosis of dynamic systems from timed observations

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ABSTRACT

It is now well-known that the size of the model is the bottleneck when using model-based approaches to diagnose complex systems. To answer this problem, decompositional and multi-modeling approaches have been proposed. In this paper, we propose a multi-modeling method called TOM4D (Timed Observations Modeling for Diagnosis) able to cope with dynamic aspects. It relies on four models: perception, structural, functional and behavior models. The behavior model is described through system component models as a set of component behavior models and the global diagnosis is computed from the component diagnoses (also called local diagnoses).

Another problem, which is far less considered, is the size of the diagnosis itself. However, it can also be huge enough, especially when dealing with dynamic system. To solve this problem, we propose in this paper to use The Timed Observation Theory.

In this context, we characterize the diagnosis using TOM4D and the timed observation theory. We show their relevance to get a tractable representation of diagnosis. To illustrate the impact on the diagnosis size, experimental results on a hydraulic example are given.

1 INTRODUCTION

Diagnosis is concerned with the development of algorithms and techniques to determine why a correctly designed system does not work as expected. The computation is based on observations, which provide information on the current behavior. The aim of diagnosis is to detect and identify the reason for any unexpected behavior, and to isolate the parts which fail in a system.

The systems to be diagnosed can be of different types, like static or dynamic systems, or systems that work with discrete or continuous domains. Moreover, the information of the system, from which the diagnosis is performed, can be qualitative, logic or quantitative. The diagnoser depends essentially on the manner in which (i) the observations are presented (ii) the system is modeled. In fact, in dynamic systems, the observation is timed unlike in static systems where the observations are given at only one point of time. The Timed Observation Theory of Le Goc (Le Goc, 2006) provides a general mathematical framework for modeling dynamic processes from timed data. This theory is important because it can be applied to all observed systems. The extension of this framework has given birth to a modeling approach for diagnosis based on timed observation theory called TOM4D. The aim of the modeling approach is to have an efficient diagnosis based on the constructed models.

For complex and large systems, the impossibility of defining a global behavior model makes it necessary to build the behavior model by breaking down and describing the behaviors of each component of the system. We extend the TOM4D method to cope with decompositional approach. In this case, diagnoses are computed locally for each component before being merged to obtain a global diagnosis.

In this paper, after a brief presentation of the Timed Observation Theory and the TOM4D method (Sections 2 and 3), we show how TOM4D supports the modeling of complex physical systems. In section 4, we show how the models can be used to characterize the diagnosis. Then, we demonstrate that the diagnosis can be computed from the TOM4D models. We apply the modeling approach and the diagnosis algorithm to an hydraulic system. Section 5 provide an application of our approach to the hydraulic system. Finally, Section 6 provides conclusions and proposes some perspectives of this work.

2 TIMED OBSERVATION THEORY

This theory defines a dynamic system as a process $Pr(t)$=$\{x_1(t),\ x_2(t),\ ...,\ x_n(t)\}$ of timed functions $x_i(t)$ defined on $\mathbb{R}$ (i.e. signals provided by sensors). A timed observation is a couple (δ^i, t_k) which corresponds to the assignation of a predicate $\theta(x_i,\ \delta^i_j,\ t_k)$ where δ^i_j is constant and $t_k \in \mathbb{R}$ a time stamp. When making an abuse of language, such a predicate can always be interpreted as the predicate EQUALS(x_i, δ^i_j,

t_k) (i.e. $x_i(t_k) = \delta_j^i$). A monitoring program $\Theta(X, \Delta)$ is a program Θ that analyzes the set of time functions x_i(t) associated to the set of variables $X = \{x_i\}_{i=1,\ldots,n}$. The aim of a monitoring program is to write timed observations (δ^i, t_k) in a database whenever a time function $x_i(t) \in$ X(t) satisfies some predicate θ(.. ,.. ,..). This leads to define the Observation class as follows:

Definition 1 (An observation class)**.** *Let X be a set of variable names of a process $Pr(t)$=$\{x_1(t), x_2(t),$ ..., $x_n(t)\}$ and let $\Delta = \bigcup_{x_i \in X} \Delta_{x_i}$ be such that Δ_{x_i} is a set of values assumable by the variable $x_i \in X$ via a program Θ. An observation class C^i=$\{(x_i, \delta_j^i), (x_{i+1}, \delta_{j+1}^{i+1})$, ..., $(x_{i+n}, \delta_{j+n}^{i+n}\}$ is a set of couple (x_i, δ_j^i) associating a variable x_i, eventually unknown, with a constant δ_j^i.*

In other words, an observation class C_i associates variables $x_i \in X$ with constants $\delta_j^i \in \Delta_{x_i}$. For simplicity reasons, an observation class is usually defined as a singleton $C_i = \{(x_i, \delta_j^i)\}$.

Proposition 2.1. *is immediate consequences of definition 1 : each timed observation o(t_k)$\equiv$ (δ_j^i, t_k) corresponds to an occurence of an Observation class $C_i = \{(x_i, \delta_j^i)\}$.*

3 MODELING FRAMEWORK FOR DIAGNOSIS : TOM4D

3.1 General Presentation of TOM4D

TOM4D is a modeling method for dynamic systems focused on timed observations. The objective of this method is to produce a suitable model for dynamic process diagnosis from timed observations and experts a priori knowledge. TOM4D relies on the idea that experts use an implicit model to both formulate the knowledge about the process and diagnose it. It is a multi-model approach that combines CommonKads templates (Schreiber *et al.*, 2000) with the conceptual framework proposed in (Zanni *et al.*, 2006) and the tetrahedron of states (T.o.S), (Rosenberg and Karnopp, 1983), (Chittaro *et al.*, 1993). These elements are merged according to the Timed Observations Theory (Le Goc, 2006).

These concepts of component, variable and observation class allow to organize the available knowledge about a process $Pr(t)$ according to a Structural Model $SM(Pr(t))$ defining the components of the process and their relations, a Functional Model $FM(Pr(t)$ defining the values of the process variables (i.e. their definition domain) and the relations between the variables values with a set of mathematical functions, and a behavior Model $BM(Pr(t))$ defining the timed observation classes firing the evolutions of the time functions of $Pr(t)$. A complementary model, the Perception Model $PM(Pr(t))$ of the process, specifies the process variables, the operating goals and the normal and abnormal operating behaviors (cf. Figure 1). Consequently, a model $M(P(t))$ of a process is a quadruplet $M(Pr(t)) = < PM(Pr(t))$, $SM(Pr(t))$,$FM(Pr(t))$, $BM(Pr(t)) >$.

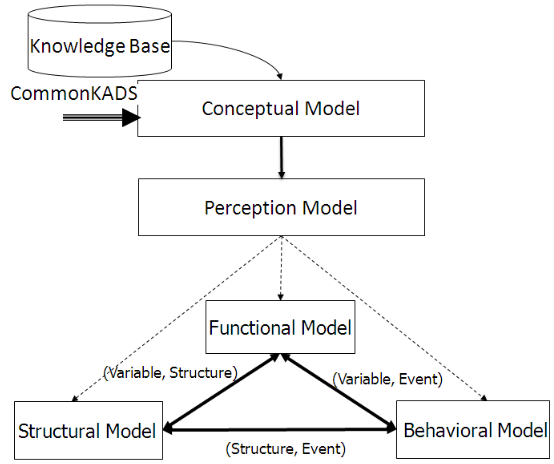

Figure 1: TOM4D Modeling Process

3.2 Improving representation in a decompositional Approach

Real-world systems can often be seen as a set of interconnected components. Each component has a simple behavior but the connections between the components can lead to a complex global model. For this reason, the size of a global model of the system is generally intractable and no global model can be effectively built. The TOM4D method associates a variable x_i^k with one and only one component c_k. This allow to decompose the dynamic process Pr as a set of subprocess Pr_k representing the different variables of component system c_k. In the following, we give the definitions of component and the properties the set of component models must satisfy to get a safe representation of the system model.

Definition 2 (system and components)**.** *A system can be described by its set of components COMPS = $\{c_1$, ..., $c_n\}$. Each component c_k is defined as a subprocess $Pr_k(t)$=$\{x_1^k(t), x_2^k(t),$ ..., $x_m^k(t)\}$ of timed functions $x_i^k(t)$. A system is viewed as a set of sub-process of $Pr(t)$ = $\{Pr_1(t)$, ..., $Pr_n(t)\}$*

The representation of the Timed Observation Theory with the decompositional system is summarized in Figure 2.

The idea is to describe the TOM4D approach at two levels namely the components of the system (component representation) and the system considered as a whole (system representation). The following sections provide a succinct description of the TOM4D approach at two levels (component and global).

3.3 Model of a component

A model of the component c_k is a quadruplet $M(Pr_k(t))$ of a process $Pr_k(t)$: $M(Pr(t))$ = $< CPM(Pr_k(t))$, $CSM(Pr_k(t))$,$CFM(Pr_k(t))$, $CBM(Pr_k(t)) >$, where CPM, CSM, CFM, CBM are respectively component perception model, component structural model, component functional model and component behavior model.

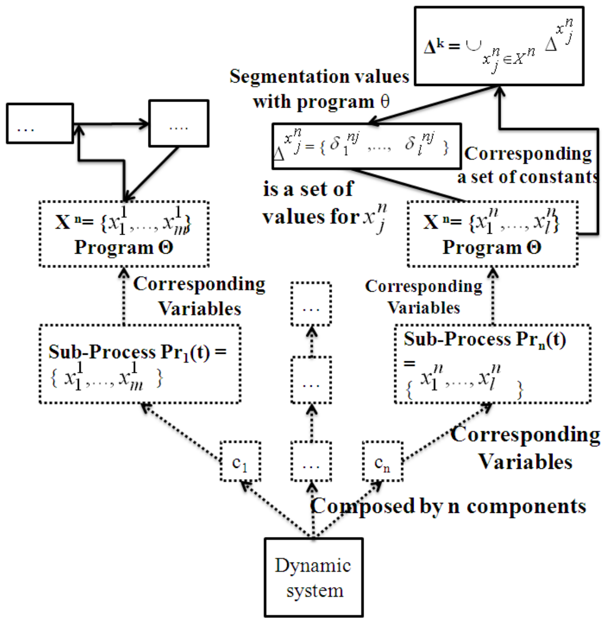

Figure 2: Timed Observation Theory extend to decompositional approach

Component Structural Model
The interpretation of the knowledge base with the T.O.S (Tetrahedron of States) allows to define an abstract generic representation of the component. The abstract model represent the structural model of the component.

Component Functional Model
The Component Functional Model describes the relations between the values of the variables of the component c_k with mathematical functions.

Component behavior Model
The component behavior Model $BM(Pr_k(t))$ of a sub-process $Pr_k(t)$ describes its operating modes with a set of states and observation classes triggering the state transitions. The behavior model is the key component of the multimodeling approach, notably because the diagnosis reasoning process is based on this model. The behavior model is defined as follow:

Definition 3 (Component behavior Model)**.** *Let X^k be a set of observable variables of a component c_k. A component behavior model $CBM(Pr_k(t))$ of this one is a 3-tuple $< S^k, C^k, \gamma_k >$ such that,*

- *S^k is a set of system states defined as $S^k = \{s^k$: $X^k \rightarrow \Delta^{x_i^k}$ as $s^k(x_i^k) = \delta^{x_i^k}$, $x_i^k \in X^k$, $\delta^{x_i^k} \in \Delta^{x_i^k}\}$,*
- *C^k is a set of observation classes where an observation class is a set of discrete events; in particular, an observation class associated with a variable $x_i^k \in X^k$ is a set $C^{x_i^k} = \{ (x_i^k, \delta)$ as $\delta \in \Delta^{x_i^k}\}$,*
- *γ_k: $S^k \times C^k \rightarrow S^k$ is a function of state transition.*

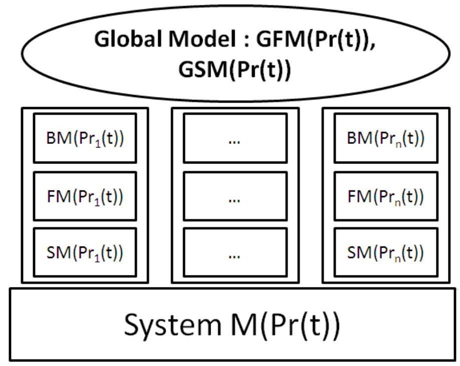

Figure 3: TOM4D presented in decompositional approach

3.4 Model of a System

The model of a system is $M(Pr(t)) = < M(Pr_1(t))$, ...,$M(Pr_n(t))$, $FGM(Pr(t)), SGM(Pr(t)) >$, where $M(Pr_k(t)) = < CPM(Pr_k(t))$, $CSM(Pr_k(t))$, $CFM(Pr_k(t))$, $CBM(Pr_k(t)) >$ is the Process Model of the component c_k, $SGM(Pr(t))$ is the global structural model and $FGM(Pr(t))$ is the global functional model. The global model $M(Pr(t))$ is built to be a decomposition of the component models $M(Pr_k(t))$. The relations between the different component models $M(Pr_k(t))$ is provided by the variables and defined in the Global Functional Model and the Global Structural Model (cf. Figure 3).

Global Structural Model
The Global Structural Model describes relations between the components of the system and the relations between the components and the variables.

Global Functional Model
The Functional Model describes the relations between the values of the variables of different components with mathematical functions.

4 DIAGNOSIS

4.1 Diagnosis characterization

This section provides a characterization of diagnosis in terms of TOM4D method and Timed Observations Theory. The system we consider is decompositional systems composed of n components (represented by n behavior models) which interact each other. We start by characterizing the diagnosis for one component then we generalize the algorithm for the whole of the system.

Each components behavior model $CBM(Pr_k(t))$ is described as a set of states and the possible transitions between them. A state transition is triggered by the occurrence of a timed observation : according to the Timed Observations Theory, such an occurrence is recorded when a variable assumes a new value. A state path (S-Path) of the component c_k between two states s_i^k and s_j^k is a suite $(s_i^k, s_{i+1}^k, ..., s_j^k)$ of $(j-i+1)$ states linking the initial state s_i^k to the final state s_j^k in a behavior model $CBM(Pr_k(t))$. This fact leads to the following definition of a state path in a behavior model:

Definition 4 (S-Path)**.** *Let $CBM(Pr_k(t))$ = $\prec S^k$, C^k, $\gamma_k \succ$ a behavior model of the process $Pr_k(t)$ representing the component c_k. A State Path is a suite $S^k_{i,j}$ = $(s^k_i, s^k_{i+1}, ..., s^k_j)$ of $(j-i+1)$ states linking the initial state s^k_i to the final state s^k_j.*

The observations ω can generally be deccomposed as follows: $\omega = \{\omega_1, . . . ,\omega_n\}$ such that ω_k contains the timed observations from the component c_k. Local diagnosis for the component c_k is performed starting from a set of local timed observations ω_k and the component behavior model $CBM(Pr_k(t))$ of the component c_k defined in TOM4D method. It consists in explaining the observations sent by the ω_k during the period $[t_0, t_n]$. The set of S-Path consistent with the $BM(Pr_k(t))$ represents the possible states occupied by the component during the period $[t_0, t_n]$. Consequently, the local diagnosis is represented by a set of S-Path, each S-Path of which represents a possible order of states of the component c_k in different instances $t \in [t_0, t_n]$.

Definition 5 (Local diagnosis definition)**.** *Given the component Behavior Model $CBM(Pr_k(t))$ of the component c_k and the local observations $\omega_k = \{o(t_0), ..., o(t_n)\}$ contains $n+1$ timed observations from the component c_k recorded during the period $[t_0, t_n]$ a diagnosis $D_{c_k}(t) = \{S^k_{i,j}\}$ is the mininal set of S-Path $S^k_{i,j}$ consistent with $CBM(Pr_k(t))$ and ω_k.*

$$\omega_k, CBM(Pr_k(t))) \rightarrow D_{c_k}(t) \quad (1)$$

These diagnoses represent the component behaviors that are consistent with the local observations. Only the S-Path leading the system to explain all the observations timed are those of interest for our local diagnosis purpose.

As a consequence, the global diagnosis $D(t)$ is a combination of each local diagnosis $D_{c_i}(t)$ at different instance of diagnosis:

Definition 6 (Global diagnosis definition)**.** *Given a set of n local diagnosis $\{D_{c_1}, ..., D_{c_k}, ..., D_{c_m}\}$ describing the possible states occupied by the different components c_k during the period $[t_0, t_n]$, noted S^k. A global diagnosis D for the system is the set of state paths S = $S^1 \times .... \times S^m$ consistent with the TOM4D models.*

$$D_{c_1}(t), ..., D_{c_m}(t), GFM(Pr(t)), GSM(Pr(t)) \rightarrow D(t)$$

In other words, the global diagnosis correspond exactly to the Cartesian product of the different possible states of the components at different instance and consistent with the global functional and global structural model of the system.

A dynamic system must continuously operate in the face of changing conditions. In the context of diagnosis, the dynamic system provides continuously new observations for its variables. According to the Timed Observation Theory, a new observation correponds to the appearance of a timed observation. Each timed observation corresponds to an occurence of an Observation class. A change of state is determinated by the occurrence of an observation class, that is to say when a variable assumes a new value.

As a consequence, the diagnosis $D(t_k)$ reasoning process must be triggered by each timed observation of a suite $\omega = \{o(t_0), ...,o(t_n)\}$ during the period $[t_0, t_n]$ in order to produce the minimal set of possible state paths that are consistent with the timed observations $o(t_k) \in \omega$. We will propose an incremental procedure to compute a diagnosis for a system description (M(Pr(t)), ω) and its successive new timed observations according to the presented definitions.

4.2 Diagnosis Algorithm

The algorithm to compute a diagnosis given a TOM4D model $M(Pr(t))$ and a suite $\omega = \{o(t_0), ...,o(t_n)\}$ of $n+1$ timed observations recorded during the period $[t_0, t_n]$ is graphically represented in Fig 4.

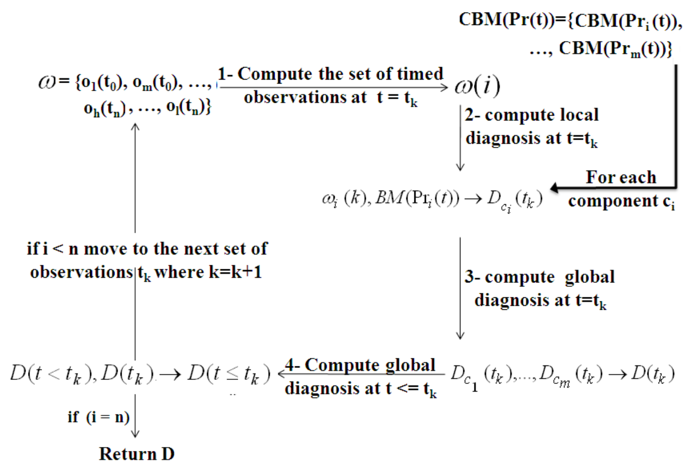

Figure 4: principle of calculating the global diagnosis of a dynamic system decomposed

Step 1 : Compute the set of timed observations at $t = t_k$

The first step of the algorithm applies the supperposition theorem of the Timed Observation Theory for each time $t_k \in [t_0, t_n]$. This theorem allows to decompose the suite ω in a set $\{\omega_1, ..., \omega_2, ..., \omega_m\}$ of m suites ω_i of (local) timed observations where each ω_i corresponds to the set of the timed observations of a component c_i.

Step 2 : Compute Local Diagnosis at $t = t_k$

The aim of the second step is to compute the local diagnosis $\omega_i(k), CBM(Pr_i(t)) \rightarrow D_{c_i}(t_k)$ for each component $c_i \in C$ at time t_k.

Step 3 : Compute Global Diagnosis at $t = t_k$

At each time t_k, the diagnosis algorithm aims then to compute each local diagnosis $D_{c_i}(t_k)$=$\{S^i_{h,m}\}$ and to combine each path sets $\{S^i_{h,m}\}$ to build a unique path set $D(t_k) = S_{i,j}$ corresponding to the global diagnosis $D(t_k)$.

Definition 7 (Global Diagnosis Composition at a time $t = t_k$)**.** *Given two local diagnosis $D_{c_i}(t_k)$ = $\{S^i_{h,m}\}$*

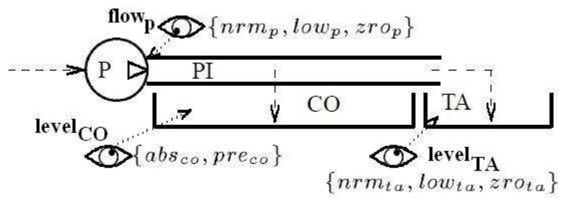

Figure 5: Hydraulic system

and $D_{c_j}(t_k)$= $\{S^j_{z,l}\}$ *(corresponds respectively to the components c_i and c_j), the global diagnosis $D(t_k)$ resulting of the combinaison of $D_{c_i}(t_k)$ and $D_{c_j}(t_k)$ is the S where: $S = S^j_{h,m} \times S^k_{z,l}$.*

This definition, that will be illustrated, can be easily generalised to any number of local diagnosis.

Step 4 : concatenation operation
Finally, to compute a global diagnosis The global diagnosis D in $[t_0, t_n]$ resulting from a sequence $\omega(k)$ of timed observations, the suite of global diagnosis $D(t_k)$ must be merge over time according to the following definition:

Definition 8 (Merging Global Diagnosis)**.** *Given two global diagnoses two diagnoses $D(t_{k-1}) = \{S^j_{n,m}\}$ and $D(t_k)$= $\{S^i_{k,l}\}$, the global diagnosis D resulting of the merging of $D(t_{k-1})$ and $D(t_k)$ is S where:*

$$\forall s \in S_{\alpha,\beta}, \exists S_{\alpha,k} \in D(t_{k-1}) and \exists S_{k,\beta} \in D(t_k)$$

$$\forall c \in C_{\alpha,\beta}, \exists S_{i,j} \in \{S^j_{n,m}\} + \{S^i_{k,l}\}$$

This last definition is used to merge all global diagnosis calculated at $t < t_k$.

5 APPLICATION CASE

We use an example to illustrate the notions mentioned above. In this section we introduce a simple device (see Fig 5) studied in (Console *et al.*, 2000) that we shall use as a running example throughout the paper.

The system is formed by a pump P which delivers water to a tank TA via a pipe PI; another tank CO is used as a collector for water that may leak from the pipe. The pump is always on and supplied of water. The pipe PI can be ok (delivering to the tank the water it receives from the pump) or leaking (in this case we assume that it delivers to the tank a low output when receiving a normal or low input, and no output when receiving no input). The tanks TA and CO simply receive water. We assume that three sensors are available (see the eyes in Figure 5): $flow_p$ measures the flow from the pump, which can be normal (nrm_p), low (low_p), or zero (zro_p); $level_{TA}$ measures the level of the water in TA, which can be normal (nrm_{ta}), low (low_{ta}), or zero (zro_{ta}); $level_{CO}$ records the presence of water in CO, either present (pre_{co}) or absent (abs_{co}).

In this paper we give the direct result for the analysis of the system with TOM4D method (more details are given in (Fakhfakh *et al.*, 2012a)). The result of modeling of the hydraulic system with the TOM4D approach means that the system is viewed as a set of sub-Process $Pr_k(t)$: $Pr(t)$ = $\{Pr_1(t)(t), Pr_2(t)(t), Pr_3(t)(t), Pr_4(t)(t)\}$ where $Pr_k(t)(t)$ = $\{x^k_1(t), x^k_2(t), x^k_3(t)\}$. These variables are determined by the analysis using the hydraulic tetrahedron of states where $x^k_1(t)$ is a volume variable (V), $x^k_2(t)$ and $x^k_3(t)$ are two outflow variables. $x^k_2(t)$ represents a normal outflow (Qs) and $x^k_3(t)$ represents an abnormal outflow corresponding to water leakage(Qf).

Table 1 shows the variable-value association and interpretations, where there is no abnormal outflow for the pump (c_1), Tank TA (c_3) and Tank CO (c_3) nor normal outflow for the c_3 and c_4.

COMPS c_k	X	dimen-sion	Value in Text	$\Delta^{x^k_j}$
c_1	x^1_1	V	nrm, low_0,zro_0	2,1,0
	x^1_2	Qs	nrm_p,low_p,zro_p	2,1,0
c_2	x^2_1	V	nrm_{pi},low_{pi},zro_{pi}	2,1,0
	x^2_2	Qs	nrm_1,low_1,zro_1	2,1,0
	x^2_3	Qf	$pres_2$,abs_2	1,2
c_3	x^3_1	V	$nrm_{TA} low_{TA}$,zro_{TA}	2,1,0
c_4	x^4_1	V	$pres_{CO}$, abs_{CO}	1,2

Table 1: component-variable-value association

Strucural Model
The component structural model CSM($Pr_k(t)$) is designed as an abstract generic hydraulic component making a relation between an input flow x^k_1, x^k_2 and x^k_3 (cf. Figure 6 a). The Global structural model $GSM(P(t))$ is a 3-tuple $< COMPS, R^p, R^x >$ (cf. Figure 6 b) where:

- $COMPS$=$\{c_1, c_2, c_3, c_4\}$ is the finite set of constants denoting the system components,
- R^p is a set of equality predicates defining the interconnections between the components. R^p=$\{out(c_1)$=$in(c_2)$, $out_1(c_2)$=$in(c_3)$, $out_2(c_2)$=$in(c_4)\}$
- R^x is a set of equality predicates linking each variable. R^x=$\{$ $out(c_1)$=x^2_1, $out(c_3)$=x^3_1, $out(c_4)$=x^4_1, $out_1(c_2)$=x^2_2, $out_2(c_2)$=$x^2_3\}$.

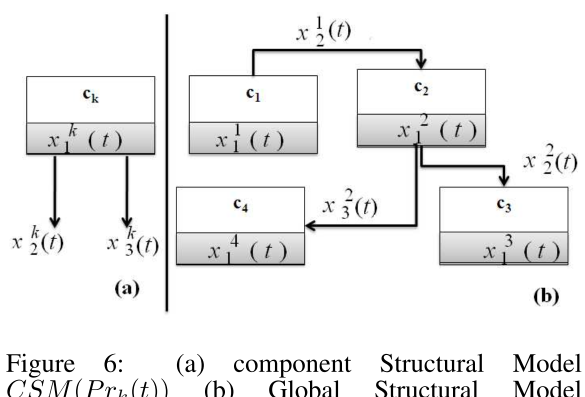

Figure 6: (a) component Structural Model $CSM(Pr_k(t))$ (b) Global Structural Model $GSM(Pr(t))$

$C_1^{x_1^1}$ = $\{(x_1^1,0)\}$	$C_2^{x_1^1}$ = $\{(x_1^1,1)\}$	$C_3^{x_1^1}$ = $\{(x_1^1, 2)\}$
$C_1^{x_2^1}$ = $\{(x_2^1,0)\}$	$C_2^{x_2^1}$ = $\{(x_2^1,1)\}$	$C_3^{x_2^1}$ = $\{(x_2^1,2)\}$
$C_1^{x_1^2}$ = $\{(x_1^2,0)\}$	$C_2^{x_1^2}$ = $\{(x_1^2,1)\}$	$C_3^{x_1^2}$ = $\{(x_1^2, 2)\}$
$C_1^{x_2^2}$ = $\{(x_2^2,0)\}$	$C_2^{x_2^2}$ = $\{(x_2^2,1)\}$	$C_3^{x_2^2}$ = $\{(x_2^2,2)\}$
$C_1^{x_3^2}$ = $\{(x_3^2,1)\}$	$C_2^{x_3^2}$ = $\{(x_3^2,2)\}$	
$C_1^{x_1^3}$ = $\{(x_1^3,0)\}$	$C_2^{x_1^3}$ = $\{(x_1^3,1)\}$	$C_3^{x_1^3}$ = $\{(x_1^3,2)\}$
$C_1^{x_1^4}$ = $\{(x_1^4,1)\}$	$C_2^{x_1^4}$ = $\{(x_1^4,2)\}$	

Table 2: The Timed Observation Classes

Functional Model

A functional model FM is a 3-tuple $< \Delta, F, R^f >$ where Δ is the set of values assumable by the different variables ($\Delta_{x_1^1}$= $\{2, 1, 0\}$for example), F is a set of functions define the relation between variables. Two types of relations are defined :

- Relation between the different variables of the same component are : $f_4(x_1^1) = x_2^1$, $f_5(x_1^2) = x_2^2$, $f_6(x_1^2) = x_3^2$ that determine the CFM;
- Relation between the different variables belonging to different components are: $f_1(x_2^1) = x_1^2$, $f_2(x_3^2) = x_1^4$, $f_3(x_2^2) = x_1^3$ that determine the GFM;

behavior Model

The set of system observation classes derived are given in Table 2 and the set of states of the pipe, for example, are represented in Table 3 (we represent only the sates physically possible using the hydraulic T.o.S). Figure 7 shows a graphic representation of the behavior model of the different components of the hydraulic system. The state transition function defines state s_2^2 as the next state when the system is in state s_1^2 and an occurrence of the $C_3^{x_1^2}$ occurs (i.e. $\gamma(s_1^2, C_3^{x_1^2}) = s_2^2$).

$States$	x_1^2	x_2^2	x_3^2	$States$	x_1^2	x_2^2	x_3^2
s_0^2	0	0	1	s_1^2	1	0	1
s_2^2	2	0	1	s_4^2	1	1	1
s_5^2	2	1	1	s_8^2	2	2	1
s_9^2	0	0	2	s_{10}^2	1	0	2
s_{11}^2	2	0	2	s_{13}^2	1	1	2
s_{14}^2	2	1	2	s_{17}^2	2	2	2

Table 3: The pipe states

Diagnosis

Let us consider the sequence of timed observation ω = $\{$ $o_{x_2^2}(t_0) \equiv$ (1, t_0), $o_{x_1^3}(t_0) \equiv$ (1, t_0), $o_{x_3^2}(t_1) \equiv$ (2, t_1), $o_{x_1^4}(t_1) \equiv$ (2, t_1), $o_{x_2^2}(t_2) \equiv$ (0, t_2) ,$o_{x_1^3}(t_2) \equiv$ (0, t_2) , $o_{x_1^2}(t_3) \equiv$ (2, t_3)$\}$ (We consider that $t_i \leq t_{i+1}$). The application of the Algorithm defined in Figure 4 following the following steps,

Step 1 : Compute the set of timed observations at $t = t_k$

The result of this step is given in table 4 (where ϕ means that there is no change of the variable value in t_i)

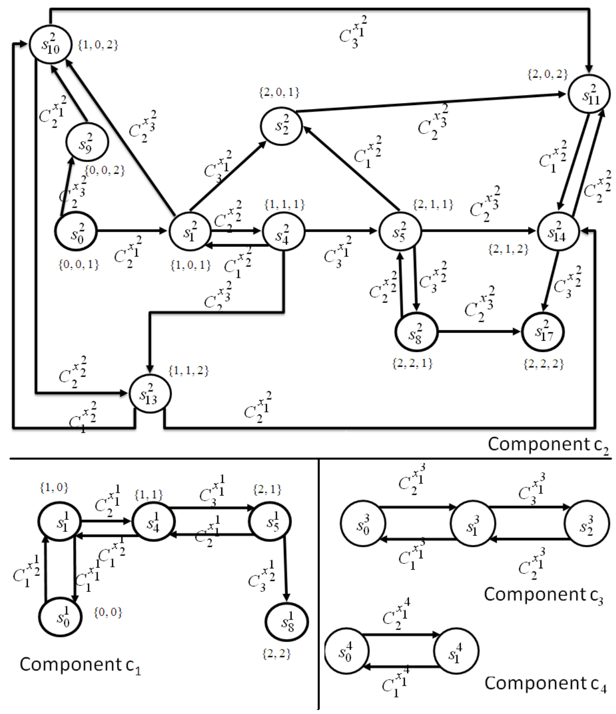

Figure 7: behavior Model of the hydraulic system

t=t_k/c_j	$\omega_1(k)$	$\omega_2(k)$	$\omega_3(k)$	$\omega_4(k)$
t_0	ϕ	$\{(x_2^2, 1)\}$	$\{(x_1^3, 1)\}$	ϕ
t_1	ϕ	$\{(x_3^2, 2)\}$	ϕ	$\{(x_1^4, 2)\}$
t_2	ϕ	$\{(x_2^2, 0)\}$	$\{(x_1^3, 0)\}$	ϕ
t_3	ϕ	$\{(x_1^2, 2)\}$	ϕ	ϕ

Table 4: Cutting observations

Step 2 : Compute Local Diagnosis at $t = t_k$

Table 5 gives the local diagnosis of c_1, c_2, c_3 and c_4 components whose TOM4D component behavior Model are respectively denoted $CBM(Pr_1(t))$, $CBM(Pr_2(t))$, $CBM(Pr_3(t))$ and $CBM(Pr_4(t))$.

For example, let us consider the pipe (c_2) component. At $t = t_0$, the x_2^2 variable assumes the new value 1 marking a state transition in the pipe. Considering its Behavior Model $CBM(Pr_2(t))$ (cf. figure 7), the possible states of c_2 after the occurence of a the class $C_2^{x_2^2}$ are defined with the state vector ($x_1^2 = \phi, x_2^2 = 1, x_3^2 = \phi$) (where ϕ denote any value), that is to say s_4^2, s_5^2, s_5^2, s_{13}^2 and s_{14}^2. Because only these states have an input arrow labelled with the class $C_1^{x_2^2}$, the possible state before the occurrence of a timed observation of this class are respectively s_1^2, s_2^2, s_8^2, s_{10}^2 and s_{11}^2. As a consequence, the possible State path (and the local diagnosis) for the c_2 component at time $t = t_0$ are $D_{c_2}(t_0)$ = $\{$ (s_1^2, s_4^2), (s_2^2, s_5^2), (s_8^2, s_5^2), (s_{10}^2, s_{13}^2), (s_{11}^2, s_{14}^2)The same reasoning is made at each time and for each component.

Dc_i (t_k)	$CBM(Pr_1$ $(t))$	$CBM(Pr_2$ $(t))$	$CBM(Pr_3$ $(t))$	$CBM(Pr_4$ $(t))$
t_0	ϕ	$D_{c_2}(t_0) = \{ (s^2_1, s^2_4), (s^2_2, s^2_5), (s^2_8, s^2_5), (s^2_{10}, s^2_{13}), (s^2_{11}, s^2_{14})\}$	$(s^3_2, s^3_1), (s^3_0, s^3_1)$	ϕ
t_1	ϕ	$D_{c_2}(t_1) = \{ (s^2_2, s^2_{11}), (s^2_8, s^2_{17}), (s^2_4, s^2_{13}), (s^2_1, s^2_{10}), (s^2_9, s^2_0)\}$	ϕ	$D_{c_4}(t_1) = \{ (s^4_0, s^4_1) \}$
t_2	ϕ	$D_{c_2}(t_2) = \{ (s^2_5, s^2_2), (s^2_{11}, s^2_{14}), (s^2_4, s^2_1) \}$	$D_{c_3}(t_2) = \{ (s^3_1, s^3_0) \}$	ϕ
t_3	ϕ	$D_{c_2}(t_3) = \{ (s^2_1, s^2_2), (s^2_{10}, s^2_{11}) \}$	ϕ	ϕ

Table 5: Local Diagnosis for different component

Step 3 : Compute Global Diagnosis at $t = t_k$
In the running example, at $t = t_0$, the possible states of the components are given in Table 6. No information

$COMPS$ $COMPS$	possible state occupied at $t < t_0$	possible state occupied at $t = t_0$
c_1	ϕ	ϕ
c_2	$s^2_1, s^2_2, s^2_8, s^2_{10}, s^2_{11}$	$s^2_4, s^2_5, s^2_5, s^2_{13}, s^2_{14}$
c_3	s^3_1	s^3_1
c_4	s^4_0	s^4_0

Table 6: Possible states occupied by the different components at t $\in]t_0, t_1]$

about the c_1 state. The possible state occupied by c_1 is the set of possible states of c_1. The global diagnosis $D(t_1)$ is concerned with 4 × 4 × 2 × 1 = 32 possible paths of different components. We use the functional model to eliminate the physically impossible states and to ensure consistency between the S-Paths calculated for example, only s^1 (x^1_2) = s^2 (x^2_1) are kept (f_4 defined in the functional model as the identity function (∀ t, $x^1_2(t)$ = $x^2_2(t)$). Only the state s^1_4 is consistent with the possible states of c_1 at t_0. Doing so lead we define the global diagnosis $D(t_i)$ in different instance for example D(t_0)= $\{ (s^1_4 \wedge s^2_4 \wedge s^3_1 \wedge s^4_0) \vee (s^1_4 \wedge s^2_5 \wedge s^3_1 \wedge s^4_0$, $(s^1_4 \wedge s^2_{13} \wedge s^3_1 \wedge s^4_0)$, $(s^1_4 \wedge s^2_{14} \wedge s^3_1 \wedge s^4_0)$ $\}$

Step 4 : concatenation operation
The definition 8 is used to merge all global diagnosis calculated at $t < t_k$. The final global diagnosis resulting the concatation of D(t_i) a different instance : D($t < t_0$), D(t_0),D(t_1),D(t_2) → D(t_3) is D(t_3) = $\{(s^1_1 \wedge s^2_{11} \wedge s^3_0 \wedge s^4_1)\}$

6 CONCLUSION

We have shown that Timed Observation Theory (and TOM4D in particular) are suitable techniques for model-based diagnosis and for studying diagnostic properties of dynamic systems. To the best of our knowledge, similar approaches have been considered in (Cordier and Grastien, 2007), (Baroni *et al.*, 1999) and (Console *et al.*, 2000) . A Comparison between PEPA, D.E.S approach and TOM4D is made in (Fakhfakh *et al.*, 2012b) . What we presented can be regarded as a preliminary work since we have not exploited all the potentialities of the theory. In particular, we will investigate how more complex properties concerning diagnosis can be defined and verified within our approach.

A problem can be discussed when we have two timed observations corresponding to one component are simultaneous (came on the same instance t_k). In fact, two timed observations are associated to two observation classes in the same instance. Consequently, the consistance with the CBM and the set of observations (such that we have defined in the article) is not possible. Hence, another extension of our algorithm is important to support the timed observations simultaneous.

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